

Reliability Engineering – Part 12

Reliability Modeling and Prediction (A)

Reliability modeling is a technique used to evaluate and predict the reliability of products and systems, including their components and sub-systems. As the name indicates, the purpose of reliability predictions is to “predict” future performance of new products and/or systems. For predictions to be reliable and meaningful they must be based on data obtained from a variety of test programs and actual performance experience.

Reliability Data

Internal Sources

In general the best data needed to perform reliability modeling and prediction are obtained from internal sources such as design evaluation testing, reliability testing, manufacturing testing, and field performance data. Also test data provided by suppliers are very useful.

External Sources

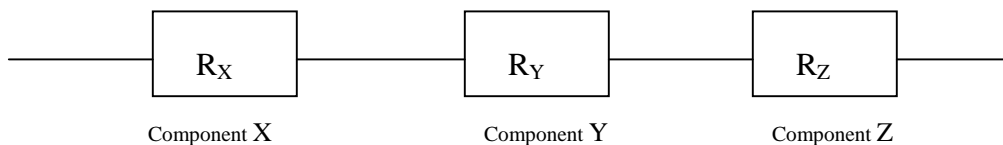
External sources include industry data and public sources (standards, literature, internet, etc.). Reliability data can also be obtained from government sources such as the Government-Industry Data Exchange Program (GIDEP), NASA, and the Reliability Information Analysis Center (RIAC).

Reliability Models

From a reliability point of view most products and/or systems can be modeled as a series, parallel, stand-by system, or combinations thereof. A **system** is defined as a functional piece of equipment consisting of a physical assemblage of parts or a process or procedure for rendering a service.

Series Systems

In a series system the components are connected in such a way that if any one of the components fails the entire system fails. See fig 11-1 below.



Note that the connections between the blocks in the diagram depict system reliability connections, not actual physical (electrical, mechanical, etc.) connections.

R_X denotes the reliability of component X. It is the probability $P(X)$ that component X functions as specified. Similarly:

R_Y denotes the reliability of component Y. It is the probability $P(Y)$ that component Y works.

R_Z denotes the reliability of component Z. It is the probability $P(Z)$ that component Z works.

The reliability of the system is the probability that the whole system works. This is equal to the probability that component X and component Y and component Z work. Assuming that the components are **independent** (meaning the failure or non-failure of a component has no bearing on the chance of failure of the other components) we can express this mathematically as follows:

$$R_{\text{system}} = P(\text{the whole system works}) = P(X \cap Y \cap Z) = P(X) * P(Y) * P(Z) = (R_X) * (R_Y) * (R_Z)$$

Ref: Conditional Probability. General Rule of Multiplication. See <http://www.asqsandiego.org/Library>. Article #38: "Probability 101"

The system's reliability can also be calculated using the general addition rule of probability. This requires more extensive calculations. (See Venn Diagram, Fig 11-2)

The probability that the system does not work is equal to the probability that component X or component Y or component Z do not work. Mathematically:

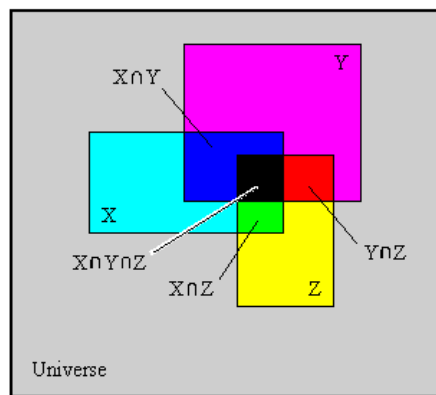
$$P(\text{system does not work}) = P(X^C \cup Y^C \cup Z^C) =$$

$$[P(X^C) + P(Y^C) + P(Z^C)] - P(X^C \cap Y^C) - P(X^C \cap Z^C) - P(Y^C \cap Z^C) + P(X^C \cap Y^C \cap Z^C) =$$

$$[P(X^C) + P(Y^C) + P(Z^C)] - \{[P(X^C) * P(Y^C)] + [P(X^C) * P(Z^C)] + [P(Y^C) * P(Z^C)]\} + [P(X^C) * P(Y^C) * P(Z^C)]$$

$$P(X^C) = 1 - P(X), \quad P(Y^C) = 1 - P(Y), \quad \text{and} \quad P(Z^C) = 1 - P(Z)$$

$$R_{\text{system}} = \text{Probability that the system works} = 1 - P(\text{system does not work})$$



Venn diagram of $X \cup Y \cup Z$

Fig 11-2

Points that belong to set X or set Y are depicted by the Cyan and Magenta areas minus the overlap between X and Y (the complete dark Blue area). Points that belong to set X or set Z are depicted by the Cyan and Yellow area minus the overlap between X and Z (the complete Green area). Points that belong to set Y or set Z are depicted by the Magenta and Yellow area minus the overlap between Y and Z (the complete Red area).

The Cyan plus Magenta plus Yellow areas minus the Blue, Green, and Red areas depict points that belong to set X or set Y or set Z. However, the three overlapping areas (Blue, Green, and Red) overlap between themselves (depicted by the Black area). Hence, we need to add the Black area back in to obtain the points in set X or Y or Z. Hence, $P(X \cup Y \cup Z) =$

$$[P(X)+P(Y)+P(Z)] - [\{P(X)*P(Y)\}+\{P(X)*P(Z)\}+\{P(Y)*P(Z)\}] + [P(X)*P(Y)*P(Z)]$$

Example 1

Let $R_X = 0.84$, $R_Y = 0.9$, and $R_Z = 0.95$ Using the multiplication rule we get:

$$R_{\text{system}} = (R_X)*(R_Y)*(R_Z) = (0.84) (0.9) (0.95) = 0.7182$$

Using the addition rule we find for the reliability of the system: $R_{\text{sys}} = 1 - F_{\text{sys}}$ in which $F_{\text{sys}} =$

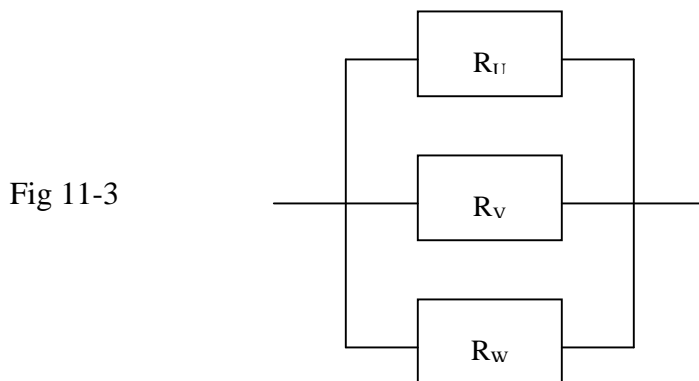
$$[P(X^C)+P(Y^C)+P(Z^C)] - [P(X^C)*P(Y^C)] - [P(X^C)*P(Z^C)] - [P(Y^C)*P(Z^C)] + [P(X^C)*P(Y^C)*P(Z^C)] =$$

$$[(0.16+0.1+0.05)] - [(0.16)(0.1)+(0.16)(0.05)+(0.1)(0.05)] + [(0.16)(0.1)(0.05)] =$$

$$0.31 - 0.029 + 0.0008 = 0.2818 \quad R_{\text{system}} = 1 - F_{\text{sys}} = 1 - 0.2818 = 0.7182$$

Parallel Systems

Figure 11-3 depicts a parallel system consisting of three elements (U), (V), and (W) that perform identical functions. As long as one of the three elements functions properly the whole system works. In other words, all three components must fail in order to experience a complete system failure. A parallel system is a particular type of redundant system. Its reliability is the probability that any one path is operational.



The reliability of the system is the probability the at least one of the three elements functions.

$$R_{\text{sys}} = \text{Probability that the system works} = P(U \cup V \cup W) =$$

$$[P(U)+P(V)+P(W)] - [\{P(U)*P(V)\}+\{P(U)*P(W)\}+\{P(V)*P(W)\}] + [P(U)*P(V)*P(W)]$$

Calculating the probability that the system does not work is a much faster and easier method for calculation the system's reliability.

$$F_{\text{sys}} = 1 - R_{\text{sys}} = P(\text{system does not work}) = P(U^C \cap V^C \cap W^C) = P(U^C) * P(V^C) * P(W^C)$$

$$R_{\text{sys}} = 1 - F_{\text{sys}} = 1 - [P(U^C) * P(V^C) * P(W^C)]$$

Example 2

Let $R_U = 0.88$, $R_V = 0.95$, and $R_W = 0.98$ The unreliability of the system is:

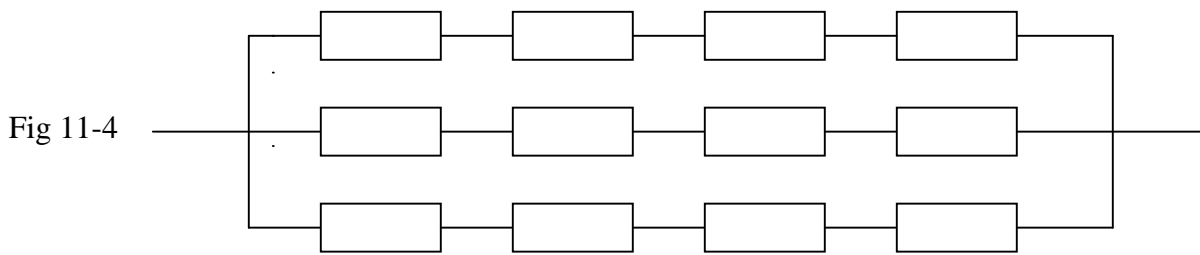
$$F_{\text{sys}} = 1 - R_{\text{sys}} = P(U^C \cap V^C \cap W^C) = P(U^C) * P(V^C) * P(W^C) = (0.12)(0.05)(0.02) = 0.00012$$

The system's reliability is: $R_{\text{sys}} = 1 - 0.00012 = 0.99988$

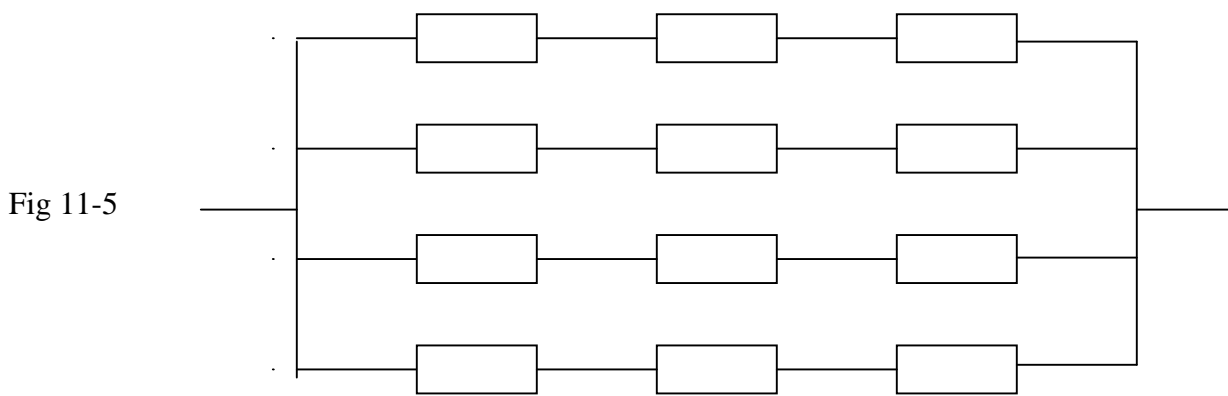
Parallel-Series, Series-Parallel, and Mixed-Parallel Systems

In many situations a system is composed of a combination of series and parallel sub-systems.

Figures 11-4, 11-5, and 11-6 show examples of combination systems.

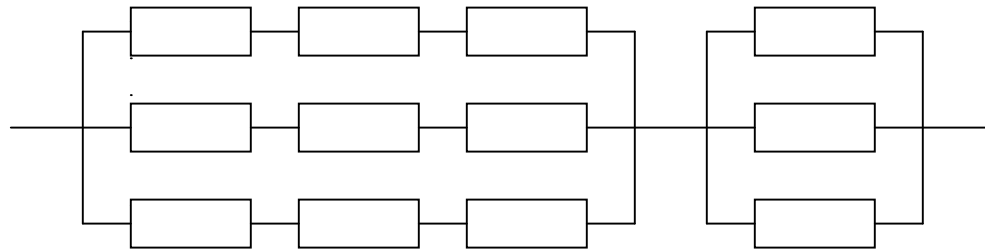


Parallel-Series System



Series-Parallel System

Fig 11-6



Mixed-Parallel System

In general, series-parallel systems have higher reliability than comparable parallel-series systems. Comparable means that both systems have an equal number of elements and each element has the same probability of failure free operation.

Example 3

Assume that the elements or sub-systems in the above systems have the same reliability of 0.90

The reliability of the Parallel-Series System is:

$$R_{\text{sys}} = 1 - \{ [1 - (0.9)^4]^3 \} = 1 - \{ [1 - 0.6561]^3 \} = 1 - (0.3439)^3 = 1 - 0.0406721 = 0.9593$$

The reliability of the Series-Parallel System is:

$$R_{\text{sys}} = 1 - \{ [1 - (0.9)^3]^4 \} = 1 - \{ [1 - 0.729]^4 \} = 1 - (0.271)^4 = 1 - 0.00539358 = 0.9946$$

The reliability of the Mixed-Parallel System is:

$$R_{\text{sys}} = [1 - \{ 1 - (0.9)^3 \}^3] * [1 - \{ 1 - 0.9 \}^3] = [1 - \{ 1 - 0.729 \}^3] * [1 - (0.1)^3] =$$

$$[1 - (0.271)^3] * [1 - 0.001] = (1 - 0.0199) (0.999) = (0.9801) (0.999) = 0.9791$$

Complex Systems

Conditional Probability Method

Some complex systems are difficult to model as a series or parallel system. Such systems can be decomposed using the **conditional probability method** (Bayes' Theorem). The method involves selecting a **keystone component**. The keystone component is usually the one that seems to link or bind together the structure of the system. It "enhances" the reliability of the system by its addition, but still allows the system to operate if it fails. The reliability of the whole system is expressed in terms of the condition of the keystone component (functioning or non-functioning).

Conditional probability is defined as $P(A|B)$. It is the probability of event (A) given that event (B) has occurred.

For any two events (A) and (B) with $P(B) > 0$, the conditional probability of (A) given that (B)

has occurred is: $P(A|B) = \frac{P(A \cap B)}{P(B)}$ and $P(A \cap B) = P(A|B) \cdot P(B)$

The conditional probability of a system failure (F) given that the keystone component is “Good”(G) is:

$$P(F|G) = \frac{P(F \cap G)}{P(G)} \quad \text{and} \quad P(F \cap G) = P(F|G) \cdot P(G)$$

The conditional probability of a system failure (F) given that the keystone component is “Bad”(B) is:

$$P(F|B) = \frac{P(F \cap B)}{P(B)} \quad \text{and} \quad P(F \cap B) = P(F|B) \cdot P(B)$$

The keystone component can be either “Good” or “Bad.” The two conditions are mutually exclusive. They cannot happen at the same time. Hence,

$$F_{\text{sys}} = P(F \cap G) \cup P(F \cap B) = P(F|G) \cdot P(G) + P(F|B) \cdot P(B)$$

The probability that a system will fail is equal to the probability that the system will fail if the keystone component is good, multiplied by the probability that the keystone component is good, plus the probability that the system will fail if the keystone component is bad, multiplied by the probability that the keystone component is bad.

Example 4

The system shown below (Fig 11-6) requires that at least on the paths is functional. The paths are: ABF, CEF, and DEF.

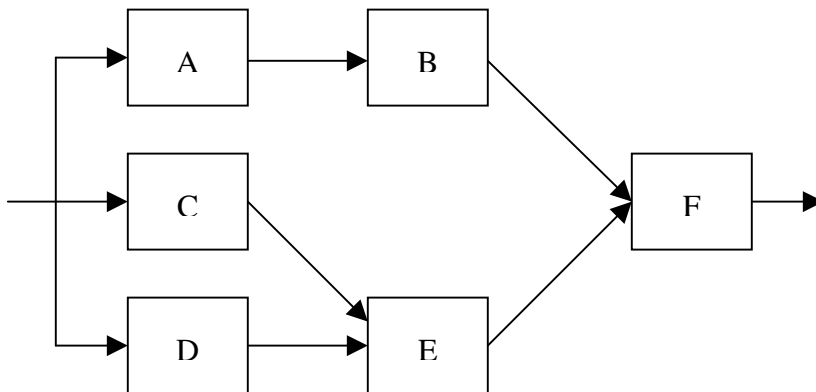


Fig 11-6

The reliability of the elements are respectively, $R_A = 0.80$, $R_B = 0.90$, $R_C = 0.75$, $R_D = 0.85$, $R_E = 0.92$, and $R_F = 0.99$

To calculate the reliability of the system we designate element E as keystone component. The reliability block diagram for $R_{\text{sys}}|E_{\text{good}}$ is shown in figure 11-7

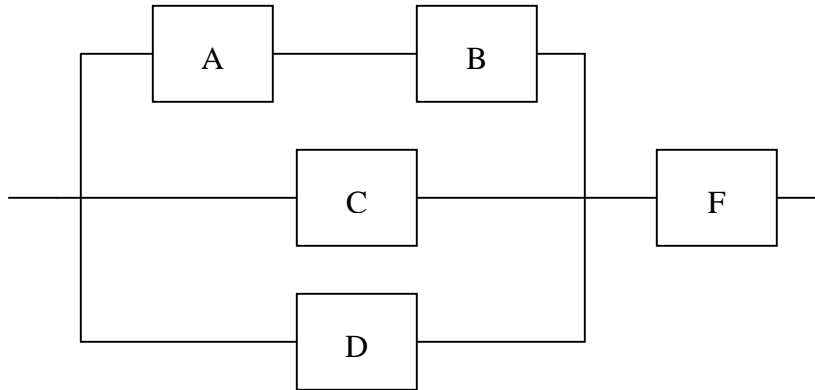


Fig 11-7

$$R_{\text{sys}}|E_{\text{good}} = [1 - \{1 - (R_A)(R_B)\} \{1 - R_C\} \{1 - R_D\}](R_F) =$$

$$[1 - \{1 - (0.8)(0.9)\} \{1 - 0.75\} \{1 - 0.85\}](0.99) = [1 - (1 - 0.72)(0.25)(0.15)](0.99) =$$

$$[1 - (0.28)(0.25)(0.15)](0.99) = (1 - 0.0105)(0.99) = (0.9895)(0.99) = 0.9796$$

The reliability diagram for $R_{\text{sys}}|E_{\text{bad}}$ is a series system consisting of elements A, B, and F (Fig 11-8)

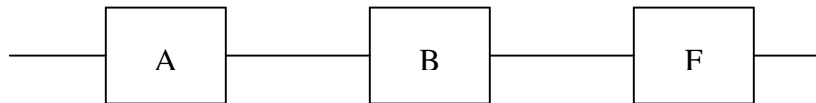


Fig 11-8

$$R_{\text{sys}}|E_{\text{bad}} = (R_A)(R_B)(R_F) = (0.8)(0.9)(0.99) = 0.7128$$

$$R_{\text{system}} = (R_{\text{sys}}|E_{\text{good}}) (R_E) + (R_{\text{sys}}|E_{\text{bad}}) (1 - R_E) = (0.9796)(0.92) + (0.7128)(0.08) = \mathbf{0.9583}$$

Cut Set and Path Set methods

We can also use the **cut set** and **path set** methods for determining the reliability of a system. These methods are usually more computational intensive as we can see in the next example.

A path set is a complete path through the reliability diagram. We don't need to determine all paths since some of the paths are contained within others. We just need to define the **minimum path sets** that do not contain any other paths within it. The system works if any one of the minimum path sets is functional. Hence, the reliability of the system is given by the union of all minimum path sets. The minimum path sets of the system in Fig 11-6 are: Path1 = ABF, Path2 = DEF, and Path3 = CEF

$$\begin{aligned}
P(\text{System works}) &= P[(A \cap B \cap F) \cup (D \cap E \cap F) \cup (C \cap E \cap F)] = \\
&P(A \cap B \cap F) + P(D \cap E \cap F) + P(C \cap E \cap F) - P(A \cap B \cap F \cap D \cap E) - P(A \cap B \cap F \cap C \cap E) - \\
&P(D \cap E \cap F \cap C) + P(A \cap B \cap F \cap D \cap E \cap C)
\end{aligned}$$

Assuming independence of probabilities of failure we get:

$$\begin{aligned}
P(\text{System works}) &= [P(A)*P(B)*P(F)] + [P(D)*P(E)*P(F)] + [P(C)*P(E)*P(F)] - \\
&[P(A)*P(B)*P(F)*P(D)*P(E)] - [P(A)*P(B)*P(F)*P(C)*P(E)] - [P(D)*P(E)*P(F)*P(C)] + \\
&[P(A)*P(B)*P(F)*P(D)*P(E)*P(C)] = (0.8)(0.9)(0.99) + (0.85)(0.92)(0.99) + (0.75)(0.92)(0.99) - \\
&(0.8)(0.9)(0.99)(0.85)(0.92) - (0.8)(0.9)(0.99)(0.75)(0.92) - (0.85)(0.92)(0.99)(0.75) + \\
&(0.8)(0.9)(0.99)(0.85)(0.92)(0.75) = 0.7128 + 0.7742 + 0.6831 - 0.55741 - 0.49183 - 0.58063 + \\
&0.4181 = 2.5882 - 1.6299 = \mathbf{0.9583}
\end{aligned}$$

Applying the **cut set** method will give an identical outcome. A **cut set** is a set of system elements that, when removed from the system, interrupts all connections between the input and output ends of the system. A minimum cut sets contains no other cut sets within it.

The minimum cut sets of the system depicted in figure 11-6 are: ACD, BCD, BE, and F. The system will fail if any one of the cut sets fails e.g. component set (A+C+D) or component set (B+C+D) or component set (B+E) or component (F).

Other methods for computing system reliability

Many systems are more complex than those reviewed in this chapter. Reliability estimation of large-scale systems such as computer, tele-communication networks or electrical power grids require computer aided simulation and modeling techniques. Some examples are:

- Boolean Truth Table Method - Factoring Algorithms
- State-Space (Markov) Analysis
- Monte Carlo Simulation

These methods are not reviewed. They are beyond the scope of this educational text.

In Part 13 we will review the reliability of **k-out-of-n** and standby systems.

References:

- Reliability Engineering – E. A. Elsayed
- Practical Reliability Engineering – Patrick O’Connor
- Applied Reliability Engineering, Vol II – Marvin Roush and Willie Web
- The CRE Primer – Robert Dovich and Bill Wortman